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Computer-based Feedback in Linear Algebra: Effects on Transfer Performance and Motivation

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Abstract

Two studies investigated the effects on students' perceptions (study 1) and learning and motivation (study 2) of different levels of feedback in mathematical problems. In these problems, an error made in one step of the problem-solving procedure will carry over to the following steps and consequently to the final solution. Providing immediate feedback after an error is made could prevent such carry-over effects. Feedback given on *all* problem-solving steps was hypothesized to yield higher motivation and better learning than feedback on the *final* problem-solving step. Study 1 investigated students' perceptions of three feedback types: 'on the *final* solution step'; 'on a*ll* the solution steps at once'; and 'on *all* the solution steps successively'. Feedback on all solutions steps was perceived by learners more positively than feedback on the final solution step. Study 2 investigated the learning and motivational effects of two types of feedback, namely, 'feedback on the *final* solution step' and 'feedback on *all* the solution steps'. The hypotheses that feedback on all problem-solving steps would lead to more effective learning and higher motivation than feedback on the final solution step were confirmed. Our results supports current efforts to implement step-wise feedback. The implications for further research and for the design of feedback are discussed.

Keywords: media in education; pedagogical issues; teaching/learning strategies

1. Introduction

Within technical domains, mathematics skills are prerequisite for most technical skills and knowledge. However, students typically have problems understanding the different representations of a mathematical function (Baki & Güvelli, 2008). If the student makes an error while solving a mathematical problem, the subsequent solution steps, and consequently the final solution, can be incorrect because the error will carry over to the following steps. To prevent this, students could be provided with feedback immediately after an error has been made (Mory, 2003). This can only be realized if the error made by the student can be detected *during* the problem-solving process and by providing specific and timely feedback to that error. The algorithmic nature of math problems makes it relatively easy to provide this type of feedback (i.e., any information given to learners about the accuracy of their response; Mory, 2003) on intermediate and final steps by a computer (Melis & Andres, 2005). However, for teachers it is nearly impossible to provide such feedback in an individualized format to more than one student simultaneously (Baki & Güveli, 2008). From that perspective, teacher provided feedback might not be the most effective way to stimulate students' learning, and consequently their motivation to continue learning.

The National Council of Teachers of Mathematics[1] (2000) report emphasizes the importance of technology for teaching and learning mathematics. Although there are many electronic tools that provide feedback after the final solution is provided by the students, only a few tools exist that allow students to solve a problem in a step-wise fashion and that provide feedback after each step (Aplusix: Nicaud, Bouhineau, & Chaachoua, 2004; LeActiveMath: Goguadze, Gonzalez Palomo, & Melis, 2005; Dutch Digital Mathematics: Boon, 2006).

This study investigated the motivational and learning effects of computer-generated feedback in linear algebra problems, either on *all* or on the *final* problem-solving step(s). It was hypothesized that feedback on *all* the problem-solving steps would yield more beneficial

[1] For more information visit the URL http://standards.nctm.org/

effects on learning and motivation than feedback on the *final* problem-solving step. Next, how feedback on the problem-solving steps can enhance students' learning and motivation as well as the role of cognitive load to interpret learning results are described.

Feedback provides students with information about their *learning* and achievement (Butler & Winne, 1995) and gives learners the opportunity to adjust and develop their cognitive strategies and to rectify misconceptions while progressing through the training (Azevedo & Bernard, 1995). Feedback has been argued to play an important role in learning and to influence performance in different ways depending on how it is provided (Hattie & Timperley, 2007). Especially during initial practice, feedback should be provided during *each* step of the problem-solving procedure (Mory, 2003). This allows learners to immediately verify the correctness of a solution step while the corresponding step is still in working memory, and it enables learners to focus their attention on the solution steps (Mory, 2003).

A student may solve an equation problem in a number of *steps*. For example, a system of linear equations can be solved by Gaussian elimination, a procedure that starts with subtracting equations from top to bottom, and then solving for the various unknowns subtracting variables from bottom to top. Gaussian elimination can be found in any standard textbook on linear algebra or matrix theory (e.g., Kolman & Hill, 2006; Lay, 2002). Feedback on *all* the problem-solving steps supports the learners to identify where (in which step) in the process something went wrong. One effective way to provide such feedback is the use of worked examples. Worked examples consist of the problem formulation, the solution steps as well as the final solution, and help learners focus their attention on problem states and problem solving operators and hence provide the opportunity to develop their own problem solving strategies (Crippen & Earl, 2007). That is, worked examples model the problem solving *process* by presenting *all* the solution steps and hence enhance understanding of the solution procedure. For novices, instruction involving worked examples yields more effective learning than instruction consisting of solving the equivalent problems (Sweller, 1988; Paas

1992; Paas & Van Merrienboer, 1994; for an overview see Atkinson, Derry, Renkl, & Wortham, 2000; Renkl, 2005). The ability to employ those parts of a learned procedure that are relevant for a new (transfer) problem requires learners' understanding of the rationale behind the different problem-solving solution steps (Catrambone, 1996). Providing feedback on the *final* problem-solving step only does not support learners to identify where exactly in the problem-solving process something went wrong, which does not contribute to the learning process. An electronic tool that recognizes the error made by the student during the problem-solving process can provide specific feedback on the step that went wrong.

Although test performance results have typically been used to assess learning outcomes, *cognitive load theory* (CLT: Paas, Renkl, & Sweller, 2003; Sweller, van Merriënboer, & Paas, 1998) argues that the imposed cognitive load to attain this performance should be considered as well. Cognitive load is commonly defined as the learner's cognitive capacity that is allocated to accommodate the demands imposed by the task, which is measured as the amount of mental effort invested by a learner to perform the task.

CLT distinguishes between three types of cognitive load: intrinsic, extraneous and germane. According to Sweller et al. (1998) intrinsic cognitive load relates to the nature of the learning material and is caused by the number of interacting information elements in a task. Besides the load imposed by the task, there is also load that is imposed by the instructional design: extraneous cognitive load (i.e., ineffective for learning) and germane cognitive load (i.e., effective for learning) (Sweller et al., 1998). From a cognitive load perspective, feedback on all the problem-solving steps might reduce extraneous or ineffective cognitive load that might be caused when students are provided with feedback on the final solution step only and have to search for a plausible explanation to the correctness or incorrectness of their solution. Similarly, worked examples are an effective way to reduce extraneous load (Paas & van Gog, 2006). The cognitive capacity that becomes available by minimizing extraneous load, for example, by step-by-step feedback or by worked examples, can be devoted to activities that contribute to learning, while inefficient strategies result in high cognitive load and low learning outcomes. Regardless of the added value of including cognitive load measures in feedback studies, to our knowledge there seem to be only a few studies (Corbalan, Kester, & van Merriënboer, 2009; Halabi, 2006) that considered the effects of feedback from a cognitive load perspective. Moreover, in those studies feedback was operationalized differently and the domains used were different from the ones used in the present study.

The effects of feedback on *motivation* have been recognized by many authors (e.g., Azevedo & Bernard, 1995; Keller, 1983; Mory, 2003; Ross & Morrison, 1993; Vollmeyer & Rheinberg, 2005). Motivated students engage in learning activities that interest them, that is, for the pleasure and satisfaction derived from their performance (Deci, Vallerand, Pelletier, & Ryan, 1991). Feedback on the problem-solving process might help learners focus on the solution steps and enables them to see the connection between what they need to learn (i.e., the steps) and the learning opportunities presented to them (Keller, 1983). This connection might eventually promote the perceived relevance of the learning material. Likewise, Schunk and Pajares (2002) suggested that timely feedback sustains motivation, and Keller and Burkman (1993) stressed that during the learning process it is very de-motivating for learners to never know how well they are performing.

The cognitive effects of motivation result primarily from the relevance of what is being learned since relevance increases the use of cognitive strategies, which will eventually improve learning (Means, Jonassen, & Dwyer, 1997). This has also been noted by Hattie en Timperley (2007) in a review of studies on feedback. The authors stated that ‘whether students engage in error correction strategies following error detection depends on their motivation to continue to pursue the goal or to reduce the gap between current knowledge and the goal’ (p. 93). That is, transfer (i.e., the ability to apply acquired knowledge and skills in new situations) will be enhanced not so much by what is being taught, but by what learners are motivated to learn themselves (Berge & Collins, 1995).

Two studies investigated the effects of different feedback types on students' perceptions (study 1) and on learning and motivation (study 2). Study 1 explored students' perceptions regarding three feedback types, namely, 'on the *final* solution step', which provided feedback on the final solution only without providing any information as to whether the intermediate solution steps were correct or not; 'on a*ll* the solution steps at once', which consisted of a worked example providing learners not only with the problem statement, but also with all the solution steps and the final solution; and 'on *all* the solution steps successively', which was a step-by-step feedback providing learners with feedback on the correctness of their responses after each problem step was solved. Study 2 measured the learning effectiveness (i.e., transfer performance), the invested mental effort, and the motivational effects of two feedback types, namely, 'feedback on the final solution step' and 'feedback on all the solution steps' (a combination of feedback on a*ll* the solution steps at once and successively in the first study).

Feedback on all solution steps was expected to support the learners to identify in which step a (potential) problem is located. In addition, more basic feedback, such as feedback on the final solution step only, does not add sufficiently to schema development, and results in a higher mental effort for its interpretation (Halabi, 2006). It was hypothesized that participants provided with feedback on all the steps of the problem-solving process would show higher effectiveness and less mental effort than participants provided with feedback on the final solution step only. In addition, feedback on all the problem-solving steps was expected to yield higher motivation, because it enables learners to see the connection between what they need to learn and the learning opportunities presented to them.

2. Study 1

2.2. Method

2.2.1. Participants

Nine first-year university students of the Technical University Eindhoven (2 females, 7 males; mean age = 18.11 years; *SD* = .60) participated. All participants spoke Dutch as their first language, the language in which the instruction was given.

2.2.2 Materials and Measurements

2.2.2.1. Electronic learning environment. Training problems were designed and delivered with Wortel TU/e [2], the electronic learning environment for mathematics available at the Technical University Eindhoven (See Figure 1 for a screenshot). The environment contained three sets of problems in linear algebra. Each set comprised three problems, increasing in difficulty, and included one of the three feedback types: ‘on the *final* solution step’, which gave information on whether the final solution was correct or wrong, followed by the correct solution; ‘on a*ll* the solution steps at once’, which consisted of a worked example showing all the solution steps at the same time; and ‘on *all* the solution steps successively’, which provided a step-by-step guide to solve the problem, more specifically, students received information about the correctness of their answer at each step and a hint to continue with the next step (see the Appendix for an example sequence). A within-subjects design was used in which all the participants received the three available feedback types. To avoid order effects, the order in which the sets and its related feedback type were provided to participants was counterbalanced.

[Insert Figure 1 about here]

2.2.2.2. Feedback specific questionnaire. The questionnaire consisted of: (a) a perception questionnaire which comprised ten statements in relation to the feedback provided (for the exact wording of each question, the reader is referred to Table 1) and it used a 5-point Likert scale ranging from ‘1 strongly disagree’ to ‘5 strongly agree’; and (b) an overall usefulness question, *'How would you rate the overall usefulness of this type of feedback?'* also measured with a 5-point Likert scale ranging from '1 very poor' to '5 excellent'. Each

[2] For more information visit the URL http://wortel.tue.nl (see 'About Wortel TU/e'). Background on the technologies can be found at Cuypers, H., Knopper, J. W., and Sterk, H. (2009). Mess: The MathDox exercise system. Electronic proceedings of the 5th Joining Educational Mathematics Workshop, Aachen, Germany.

participant filled out the questionnaire three times, one time per feedback type. Reliabilities of the perception questionnaire for the three types of feedback were .93, .90, and .83 (Cronbach's alpha) for, in order, 'feedback 1', 'feedback 2', and 'feedback 3'.

2.2.3. Feedback general questionnaire. This questionnaire consisted on two sets of questions. Set 1 contained eight multiple-choice questions which required participants to choose one of the three feedback types (e.g., *Which of the three types of feedback provided do you prefer the most?; Which of the three types of feedback was the least informative?).* For the exact wording of each question, the reader is referred to Table 2. Set 2 contained three questions in which participants had to indicate their preferred type of feedback given an easy, a medium, and a difficult linear algebra problem, respectively.

2.3. Procedure

First, one of the two experimenters explained the purpose of the session to the participants. Subsequently, participants worked with the three sets of linear algebra problems. After each set of problems, participants filled out the 'feedback specific questionnaire', which measured their perceptions regarding the specific feedback type they just received. Each set of problems took between 15 and 20 minutes to complete. After the three sets of problems were completed, participants filled out the 'feedback general questionnaire'.

3. Results

3.1. Feedback specific. A nonparametric Friedman's test revealed that participants' perception was significantly different between the three types of feedback (X^2 (2) = 10.89, $p <$ .01). Wilcoxon tests were used to follow up this finding. After a Bonferroni correction, all effects are reported at a .0167 level of significance. Participants' perception was significantly different between the feedback 'on the *final* solution step' and the feedback 'on a*ll* the solution steps at once' (Z = -2.67, p = .01) and between the feedback 'on the *final* solution step' and the feedback 'on a*ll* the solution steps successively' (Z = -2.25, p = .015). Similarly, participants' perceived overall usefulness was significantly different (X^2 (2) = 8.06, p < .025).

Follow up Wilcoxon tests revealed significant differences between the perceived overall usefulness of students between the feedback ‘on the *final* solution step’ and the feedback ‘on a*ll* the solution steps at once’ (Z = -2.40, p = .01). No other significant effects were found.

[Insert Table 1 about here]

Feedback general. Table 2 presents the students’ selected feedback types to each question of the two sets of questions.

[Insert Table 2 about here]

Chi-square goodness-of-fit were used to analyze the results. Regarding the first set of questions, results show a significant difference between the three feedback types on ‘question 1’ (X^2 (2) = 8.67, p = .013); ‘question 2’ (X^2 (2) = 8.67, p = .013); ‘question 6’ (X^2 (2) = 18, p = .000); and ‘question 8’ (X^2 (2) = 12.67, p = .002). Additionally, marginal significant differences were found on ‘question 4’ (X^2 (2) = 6, p = .05); ‘question 5’ (X^2 (2) = 6, p = .05); and ‘question 7’ (X^2 (2) = 6, p = .05). With regard to the second set of questions, which required participants to select their preferred feedback type given a specific problem with low, medium, and high difficulty, a marginal significant effect was found on ‘question 3’ (X^2 (2) = 6, p = .05). Figure 2 depicts students’ preferred feedback type per difficult level.

[Insert Figure 2 about here]

4. Discussion

This first study gives insight into students’ perceptions regarding three different types of feedback in linear algebra problems, namely, feedback ‘on the final solution step’ and two feedback on all the solution steps: 'at once’ and ‘successively’. Students’ perceptions were clearly more positive for the two feedback types which provided feedback on all the solution steps as compared to the feedback provided on the final solution step only. Consequently, the next study investigated the motivational and learning effects of a feedback condition in which the two types of feedback on all the solution steps were combined, as compared to a feedback condition in which only feedback on the final solution step was provided. None of the students selected feedback ‘on the final solution step’ as the most preferred, the easier to follow, the most informative, or as the type of feedback that they would choose for their own learning. In addition, none of the participants selected feedback ‘on the final solution step’ for solving the most difficult linear algebra problem. The degree to which a specific feedback type is preferred to solve linear algebra problems seems to depend on the level of difficulty of the exercise to solve. It appears that students prefer more detailed feedback as the level of difficulty of the problems increases. A finding that might be interesting to investigate pertains to the fact that more students selected feedback ‘on all the solution steps at once’ as being more informative than feedback ‘on all the solution steps successively’. A possible explanation might be that learners perceive more control by immediate feedback on all solutions steps as they are free to choose in what order to look at the solution.

In this study only nine students participated. Although the design was within-subjects and results were analyzed using non-parametric tests, a replication of the study should include more participants. In addition, although this study aimed at exploring students’ perceptions of feedback, a replication of this study, which would include measures of students’ performance as well, could shed more light on the inconsistent results found in feedback studies. Forms of feedback which contain simple but sufficient information may help minimizing the associated cognitive load to process the information, because it might contain considerable less distracting information (for a review see Mory, 2003). According to this premise and to students’ selections, for simple problems a more basic feedback form (e.g., feedback ‘on the final solution step’) may prove sufficient. In sum, results show that students clearly prefer feedback on all the solution steps to feedback on the final solution step only.

5. Study 2

5. 2. Method

5. 2.1.Participants

Thirty-four university students of the Technical University Eindhoven (5 females, 29 males; mean age = 20.29 years; *SD* = 2.98) participated. All participants spoke Dutch as their first language, the language in which the instruction was given. Because in study 1 students' clearly preferred both feedback types 'on all the solution steps' over feedback on the final solution step only, in study 2 the two types of feedback on all the solution steps were combined into one type, namely, feedback 'on all the solution steps'. Students were randomly assigned to one of the two experimental groups. In the feedback 'on the final solution step' condition ($n = 17$), the participants were provided with information on the correctness of their final solution to the problems (i.e., right/wrong). In the feedback 'on all the solution steps' condition ($n = 17$) the participants could choose whether they wanted to receive feedback on each solution step at once (worked example) or successively (step-by-step).

5. 2.2. Materials and Measurements

5.2.2.1.Training problems. As in study 1, training problems were designed and delivered with Wortel TU/e. The learning environment contained two sets of problems in linear algebra. Each participant worked on the first set during a first session and on the second set during a second session. The first set contained ten problems regarding the problem of solving a set of linear equations and on subspaces, basis and dimension of linear space (in Dutch abbreviated to, vergelijkingen en bases). The second set contained ten problems regarding matrices and linear transformations (in Dutch, matrices en afbeeldingen). In both feedback conditions, each set contained the same type of problems and followed the same sequence. Figure 3 depicts two different training tasks in the feedback 'on the final solution step' condition (2a) and in the feedback 'on all the solution steps' condition (2b). Readers interested in getting more insight into the type and content of the training tasks are referred to Cuypers (2007)[3], who describes a number of rules and strategies to solve exercises in the area of basic linear algebra.

[3] Cuypers, H. (2007). Rules and Strategies for Linear Algebra. Internal report Surf project Intelligent Feedback, available at http://ideas.cs.uu.nl/wiki/index.php/Project_documents

[Insert Figure 3 about here]

5. 2.2.2. Test problems. Test tasks were delivered on paper. In total, there were eight problems, more specifically, 4 retention tasks and 4 transfer tasks. The retention tasks were structurally similar to the training tasks (see Figure 4a for an example) and determined whether participants were able to apply the learned procedures in the same way as in the training tasks. The transfer tasks required participants to flexibly apply the learned solutions procedures to tasks with different structural features than those encountered during the training sessions (see Figure 4b for an example). For example, during the transfer test students encountered various tasks for finding the nullspace of a 2x2 or 3x3 matrix. In Figure 3b, the student had to realize, that taking derivatives on the space of differentiable functions that are linear combinations of *f, g* and *h* is a linear mapping that can be represented by a 3x3 matrix. The function *k* can be written as a linear combination of *f, g* and *h* if and only if the matrix has a nontrivial vector in its nullspace.

[Insert Figure 4 about here]

5. 2.2.3. *Mental effort.* Mental effort was used as an index for cognitive load, which refers to the amount of cognitive capacity that is allocated to problem solving. Mental effort was measured as the “effort required to solve the task” (Paas et al., 2003) after each learning task and after each transfer task with a one-item 9-point Likert-type scale ranging from 1 (very very low effort) to 9 (very very high effort). Reliability of the mental effort measures reported during the training was .89 (Cronbach’s alpha) and during the test problems .72 and .73 (Cronbach’s alpha) for the retention and transfer test, respectively.

5. 2.2.4. Motivation questionnaire. The Instructional Materials Motivation Survey (IMMS; Keller, 1983) assesses the motivational effects of instructional situations. Specifically, it asks students to rate 36 statements tapping attention (i.e., *need for stimulation and variety*), relevance (i.e., *desire to satisfy basic motives*), confidence (i.e., *desire to feel competent and in control*), and satisfaction (i.e., *desire to feel good about oneself*), about the learning materials according to the ARCS model (Keller, 1983) with four respective subscales, namely Attention (e.g., “There was something interesting at the beginning of each task that got my attention”), Relevance (e.g., “It was clear to me how the content of the tasks was related to things I already know”), Confidence (e.g., “As I worked on the tasks, I was confident that I could learn the content”), and Satisfaction (e.g., “Completing the tasks gave me a satisfying feeling of accomplishment”). The scales contained, in order, twelve, nine, nine, and six items and were measured with a 5-point rating scale ranging from 1 (completely not true) to 5 (completely true). The reliability of the motivation questionnaire was .78 (Cronbach’s alpha).

5. 2.2.5. Perception questionnaire. Participants received a similar questionnaire to the feedback specific questionnaire presented in study 1. Two additional “yes / no” questions which asked participants whether they ‘*would like to see some features added to this type of feedback’* and whether they ‘*would like to see some features removed from this type of feedback’*. Reliability of the perception questionnaire was .93 (Cronbach’s alpha).

5.3. Procedure

First, one of the experimenters explained the purpose of the experimental sessions to the participants. They were explained that they would have to work during three different sessions: During each of the two first sessions, they would have to work on the computer on a set containing 10 types of linear algebra problems, and that for each problem feedback would be provided. They were also instructed that during the third session they would have to work on a post test containing eight paper and pencil linear algebra problems and that no further feedback would be provided. Subsequently, participants filled out a question on their prior knowledge in linear algebra and started the first session. The session took a maximum of two hours. Within the same week, participants worked on the second session for a maximum of two hours. After each training problem, mental effort was measured by asking the participants to fill out the 9-point rating scale. Immediately after the second session, participants filled out the perception and the motivation questionnaires. During the training sessions, the time spent by the participants was logged. The third session also took place within the same week. After each test problem, mental effort was measured by asking the learner to fill out the 9-point rating scale. During the test phase participants were allowed to work at their own pace.

6. Results

The number of participants who reported having low, medium, and high prior knowledge on linear algebra was evenly distributed over the conditions ($X^2 = 1.98$, $p = .37$). Hence, the results reported here are not likely to be artifacts of prior knowledge differences between groups. Analyses were conducted by using one-tailed independent samples *t*-tests. Cohen's *d* is provided as an estimate of effect size, with *d*-values of .20, .50, and .80 corresponding to small, medium, and large effects, respectively (Cohen, 1988).

6.1. Training Phase

Table 3 provides an overview of the mean scores and standard deviations for the training and test variables.

[Insert Table 3 about here]

6.1.1. *Performance.* No effects on performance during the training session (sessions 1 and 2) were found. One of the experimenters, who was also the teacher who developed the problems for this study, divided the problems provided in three levels of complexity (low, medium, and high). The results of the perception test provided to students in the first test indicated that students prefer feedback on all the solution steps as the level of complexity of the problems increases. Therefore, additional analyses were carried out to investigate whether the effects on performance scores of the two types of feedback varied as a function of the complexity level of the problems. As Figure 5 depicts, for problems with a low and a medium complexity level, students performed better during training with feedback 'on the final solution'. For problems with a high complexity level, students performed better with feedback 'on all the solution steps'. However, the expected pattern found did not reach significance (all *p*'s > .05).

[Insert Figure 5 about here]

6.1.2. Mental effort. Mental effort invested during training was higher for students in the feedback 'on the final solution' condition (*M* = 4.44; *SD* = 1.18) than for students in the feedback 'on all the solution steps' condition (*M* = 3.76, *SD* = .95); *t*(32) = 1.84, *p* < .05, *d* = .36 (one-tailed).

6.2. Test Phase

6*.2.1. Test Performance.* Results on the retention test showed no significant differences between both feedback conditions. Analyses showed a significant difference between groups on transfer performance, *t*(29) = -2.22; *p* < .015, *d* = -.80 (one-tailed). Participants in the feedback 'on all the solution steps' condition scored higher on transfer (*M* = 22.65, *SD* = 12.08) than participants in the feedback 'on the final solution' condition (*M* = 13.65, *SD* = 10.48).

6.2.2. Mental effort. No effects were found.

6.3. Questionnaires

Table 4 provides an overview of the mean scores and standard deviations for the motivation and the perception questionnaires.

[Insert Table 4 about here]

6.3.1. Motivation questionnaire. Results showed a significant effect of feedback on relevance, *t*(32) = -2.75, *p* < .01, *d* = - .93; confidence, *t*(32) = -1.88, *p* < .05, *d* = -.63; and satisfaction, *t*(32) = -2.90, *p* < .001, *d* = -.99. Participants in the feedback 'on all the solution steps' condition reported higher relevance (*M* = 3.50; *SD* = .34); higher confidence (*M* = 3.84; *SD* = .23), and higher satisfaction (*M* = 3.43; *SD* = .36) than participants in the feedback 'on the final solution' condition (*M* = 3.15; *SD* = .41; *M* = 3.56; *SD* = .59; and *M* = 3.06; *SD* = .39 for relevance, confidence, and satisfaction, respectively). Analyses showed no significant differences between the two groups on attention.

6.3.2.Perception questionnaire. Results showed a significant difference between groups on the participants' perceptions of feedback, $t(31) = -8.83$, $p < .001$ $d = -3.02$. The mean in the perception scale of participants in the feedback 'on all the solution steps' condition ($M = 3.88$, $SD = .48$) was higher than the mean of participants in the feedback 'on the final solution' condition ($M = 2.16$, $SD = .66$). Similarly, analyses showed a significant difference between participants' perceived usefulness of feedback, $t(31) = -5.25$, $p < .0001$ $d = -1.84$. Perceived usefulness was significantly higher in the feedback 'on all the solution steps' condition ($M = 3.94$, $SD = .85$) than in the feedback 'on the final solution' condition ($M = 2.12$, $SD = 1.11$). Table 5 presents students' responses on the "no / yes" questions. More participants in the feedback 'on the final solution' condition (82.4%) would like to see some features added to the feedback than in the feedback 'on all the solution steps' condition (29.4%; $X^2 = 9.66$, $p = .002$).

[Insert Table 5 about here]

7. Discussion

This study investigated the effects of two types of feedback, namely, feedback 'on the final solution' and feedback 'on all the solution steps' on learning effectiveness, invested mental effort, and motivation. We hypothesized that providing learners with feedback on all the solution steps would lead to higher effectiveness. This hypothesis was supported by the findings. Learners receiving feedback on each solution step showed higher transfer performance than learners receiving feedback on the final solution only. Learners provided with feedback on each solution step possibly constructed general problem-solving skills which enabled them to flexibly apply the learned solution procedure to solve unfamiliar linear algebra problems. Furthermore, these learners also reported less mental effort during training. Presumably, they were better able to allocate their working memory capacity to processes relevant for learning. However, no differences between conditions were found for effectiveness on the retention test. The general information available in the problem-solving skills constructed, although particularly useful to deal with tasks that require learners to *flexibly* apply the learned solution procedure, as in transfer problems, could have been of less use for familiar tasks that require learners to apply the learned solution procedure similarly to the practiced problems, as in the retention problems (Sweller et al., 1998).

The hypothesis that feedback on all the problem-solving steps will motivate the students more than feedback on the final solution was validated as well. Participants who received this form of feedback scored significantly higher on three of the four scales of the IMMS (relevance, confidence, and satisfaction) perceived the feedback received as more positive and useful than participants who were provided the more basic feedback. Feedback on each solution step could have enabled students to connect what is presented to them to what they already know. Hence, results support Keller's (1983) theory of motivation, which argues that the motivation of a learner can be manipulated by instructional design.

A final finding that needs to be discussed is the fact that mental effort invested during training was relatively low (all means are below the neutral score of 5). This could indicate that in general the problems were not too complex for the learners, and that the more elaborate type of feedback could have been useful in the most complex problems. Future studies may examine the effects of elaborate and simpler forms of feedback on learners' performance when more complex problems than the one used in this study are used.

8. General Discussion

These studies aimed at investigating the effects on students' perceptions (study 1) and learning and motivation (study 2) of feedback 'on all the problem-solving steps' as compared to feedback 'on the final solution step' only. Overall, participants were more positive, performed better, and were more motivated when feedback was provided on all the solution steps than on the final solution step only. Our results also extend feedback research by applying cognitive load theory by analyzing whether feedback on all the steps of the problem-solving process reduces the invested mental effort to solve linear algebra problems.

In addition, although in the second study the analyses on performance and level of problem difficulty during training did not yield significant differences, with regard to the most difficult problems the pattern was in the expected direction: Students in the feedback ’on the final solution step’ condition scored lower (*M* = 48.72, *SD* = 32.88) than students in the feedback ‘on all the solution steps’ condition (*M* = 61.18, *SD* = 26.34). This is also in line with the students’ responses in study 1 in which they indicated to prefer step-by-step feedback (feedback on all the solution steps successively) and the worked examples (feedback on all the solution steps at once) for the most difficult linear algebra problem.

The means of the students’ perceptions of the feedback were rather similar in both studies. In general, students prefer feedback on all the solutions steps over feedback on the final solution step only. In both studies, participants filled out the questionnaire *after* receiving *all* the linear algebra problems linked to one specific feedback type. Future efforts might examine whether students’ perceptions differ as a function of the difficulty level of the problems. This could be measured by asking students to fill out the questionnaire, or possibly just a single item, *after each* task or set of tasks from a certain difficulty level. With regard to the claimed transfer effects of feedback on all solution steps, one remark should be made. The transfer test was administered in the same week than the training sessions took place. Hence, nothing can be concluded about long-term effects in this study. Future research is needed to determine whether the results can also be found with delayed assessments of transfer test performance, or whether feedback on the final solution could yield better long-term effects than feedback on all the solution steps.

These findings also provide other implications for future research. First, whether gradually increasing the level of detail of the feedback along with the difficulty level of the problem yields better results on learning and motivation could be investigated. In a feedback review, Mory (2003) argued that simple but sufficient feedback in some situations may help minimizing the associated cognitive load to process the information because it might contain considerable less distracting information. A recent study (Halabi, 2006) found that rich feedback was significantly more useful for students with no prior knowledge. Future studies may *adapt* the feedback level to each individual learner's expertise throughout the training. Third, more advanced techniques such as eye-tracking or thinking-aloud protocols may uncover learners' problem-solving processes. This might add valuable information to the learning measures used in the study and shed some light on how learners actually respond on the different feedback types. Finally, Kramarski and Zeichner (2001) found positive effects of metacognitive feedback (which used metacognitive questions that serves as cues for understanding) on performance in mathematical reasoning and explanations. Future studies might investigate the effects of metacognitive feedback on performance and motivation.

9. Conclusions

The two studies indicate that, for students of a technical domain, feedback provided on all the problem-solving steps is more effective and it is clearly preferred by learners and more motivating than feedback provided on the final solution step only. These results are particularly important because it shows the importance of the current efforts being made in developing learning tools that provide learners computer generated feedback on the problem-solving process as they progress throughout the training. More specifically, an important practical implication of this study for the design of electronic environments that offer automatically generated feedback in highly structured subjects, such a linear algebra, is that such electronic environments should incorporate step-wise guidance which support students understand the correctness or incorrectness of their solutions.

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Table 1

First Quartile, Median and Third Quartile Values of the Friedman's test (1a) and Sums of Ranks of the Wilcoxon tests (1b) for the Feedback Specific Questionnaire

	Feedback								
	Final solution			All steps at once			All steps successively		
	1st	Median	3rd	1st	Median	3rd	1st	Median	3rd
Perception scale - This type of feedback was informative - This type of feedback was stimulating - This type of feedback was interesting - I would recommend this type of feedback to other students - I would choose this type of feedback to solve another problem - The content of this type of feedback made the problem easier to solve - The level of detail of this type of feedback was appropriate for me - This type of feedback appropriately covered what I needed to know about how to solve the problem - The purpose of this type of feedback was clear - I feel confident I could successfully solve a similar problem with this type of feedback	1.25	2.60	3.15	3.45	3.70	4.50	3.85	4.20	4.60
Overall usefulness	1.00	3.00	3.50	3.00	4.00	5.00	3.50	4.00	5.00

(1a)

	Feedback					
	On all the solution steps at once / On the final solution step		On all the solution steps successively / On the final solution step		On all the solution steps successively / On all the solution steps at once	
	Positive ranks	*Negative ranks*	*Positive ranks*	*Negative ranks*	*Positive ranks*	*Negative ranks*
Perception scale	45.00	.00	41.50	3.50	29.00	16.00
Overall usefulness	28.00	.00	31.50	4.50	18.50	17.50

(1b)

Table 2

Feedback Type Selected by the Participants

	Feedback type		
	On the final solution step	On all the solution steps at once	On all the solution steps successively
Set 1: Selected feedback type			
1) Which of the three types of feedback provided do you prefer the *most*?	0	2	7
2) Which of the three types of feedback provided do you prefer the *least*?	7	0	2
3) Which of the three types of feedback was *easier* to follow?	0	4	5
4) Which of the three types of feedback was more *difficult* to follow?	6	0	3
5) Which of the three types of feedback was the most *informative*?	0	6	3
6) Which of the three types of feedback was the least *informative*?	9	0	0
7) Which of the three types of feedback would you primarily *choose* for your learning?	0	3	6
8) Which of the three types of feedback would you primarily *not* choose for your learning?	8	0	1
Set 2: Given the problem below, which type of feedback would you choose?			
1) Low difficulty	3	4	*2*
2) Medium difficulty	2	3	4
3) High difficulty	0	3	6

Table 3

Means and Standard Deviations of the Training Phase and the Test Phase

	Feedback			
	On the final solution step		On all the solution steps	
	M	*SD*	*M*	*SD*
Training Phase				
Performance	61.15	21.80	59.00	13.03
Mental Effort	4.44	1.18	3.76	.95
Test Phase				
Performance on Retention Test	32.65	10.517	35.59	4.68
Performance on Transfer Test	13.65	10.48	22.65	12.08
Mental Effort on Retention Test	3.91	1.51	3.38	1.08
Mental Effort on Transfer Test	5.92	1.70	5.68	1.59

Table 4

Means and Standard Deviations of the Questionnaires Provided after Session 2 (maximum score of 5)

	Feedback			
	On the final solution step		On all the Solution steps	
	M	*SD*	*M*	*SD*
Perception				
Perception scale	2.16	.66	3.88	.46
Overall usefulness	2.12	1.11	3.94	.85
Motivation				
Attention	3.35	.30	3.27	.35
Relevance	3.15	.41	3.50	.34
Confidence	3.56	.59	3.84	.23
Satisfaction	3.06	.39	3.43	.36

Table 5

Response Percentages of the "No / Yes" Questions per Feedback Type in the Perception Questionnaire

	Feedback type	
	On the final solution step	On all the Solution steps
Would you like to see some features *added* to this type of feedback?		
% No	17.6	70.6
% Yes	82.4	29.4
Would you like to see some features *removed* from this type of feedback?		
%No	94.1	76.5
%Yes	5.9	23.5

Figure Captions

Figure 1. Screenshot of Wortel TU/e (in Dutch).

Figure 2. Participants' preferred feedback type per difficult level (from easy to difficult).

Figure 3. Two examples depicting a task in the feedback 'on the final solution step' condition (A) and a different task in the feedback 'on all the solution steps' condition (B).

Figure 4. Example of a retention task (A) and of a transfer task (B).

Figure 5. Results of training performance depending on feedback type and complexity level (low, medium, high).

Figure 1

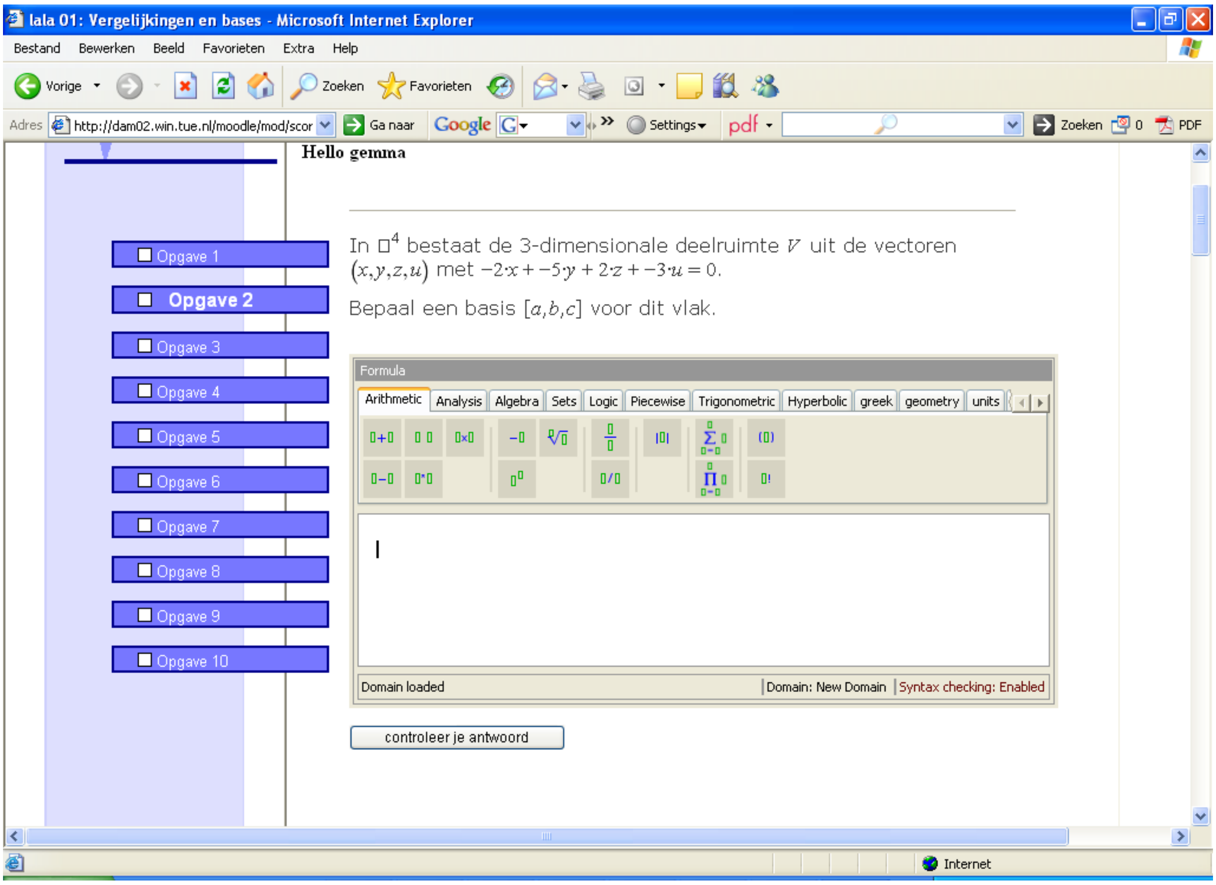

lala 01: Vergelijkingen en bases - Microsoft Internet Explorer
Bestand Bewerken Beeld Favorieten Extra Help
Vorige
Zoeken
Favorieten
Adres http://dam02.win.tue.nl/moodle/mod/scor
Ga naar
Settings
pdf
Zoeken
PDF
Hello gemma
Opgave 1
Opgave 2
Opgave 3
Opgave 4
Opgave 5
Opgave 6
Opgave 7
Opgave 8
Opgave 9
Opgave 10
In □⁴ bestaat de 3-dimensionale deelruimte V uit de vectoren (x,y,z,u) met −2·x + −5·y + 2·z + −3·u = 0.
Bepaal een basis [a,b,c] voor dit vlak.
Formula
Arithmetic
Analysis
Algebra
Sets
Logic
Piecewise
Trigonometric
Hyperbolic
greek
geometry
units
Domain loaded
Domain: New Domain
Syntax checking: Enabled
controleer je antwoord
Internet

Figure 2

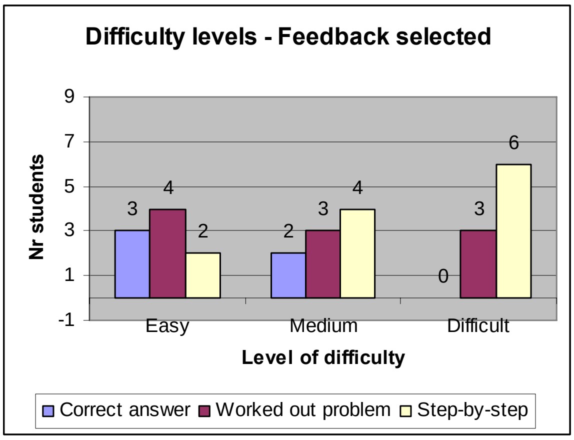
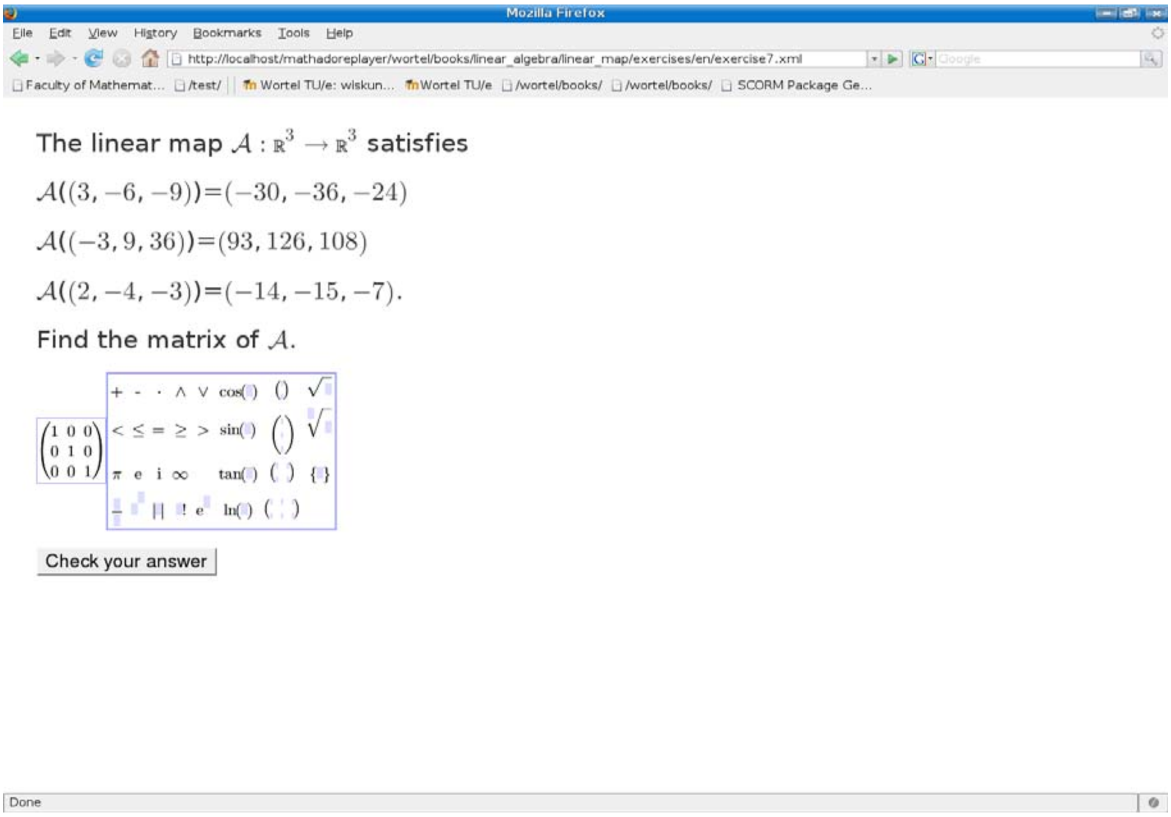
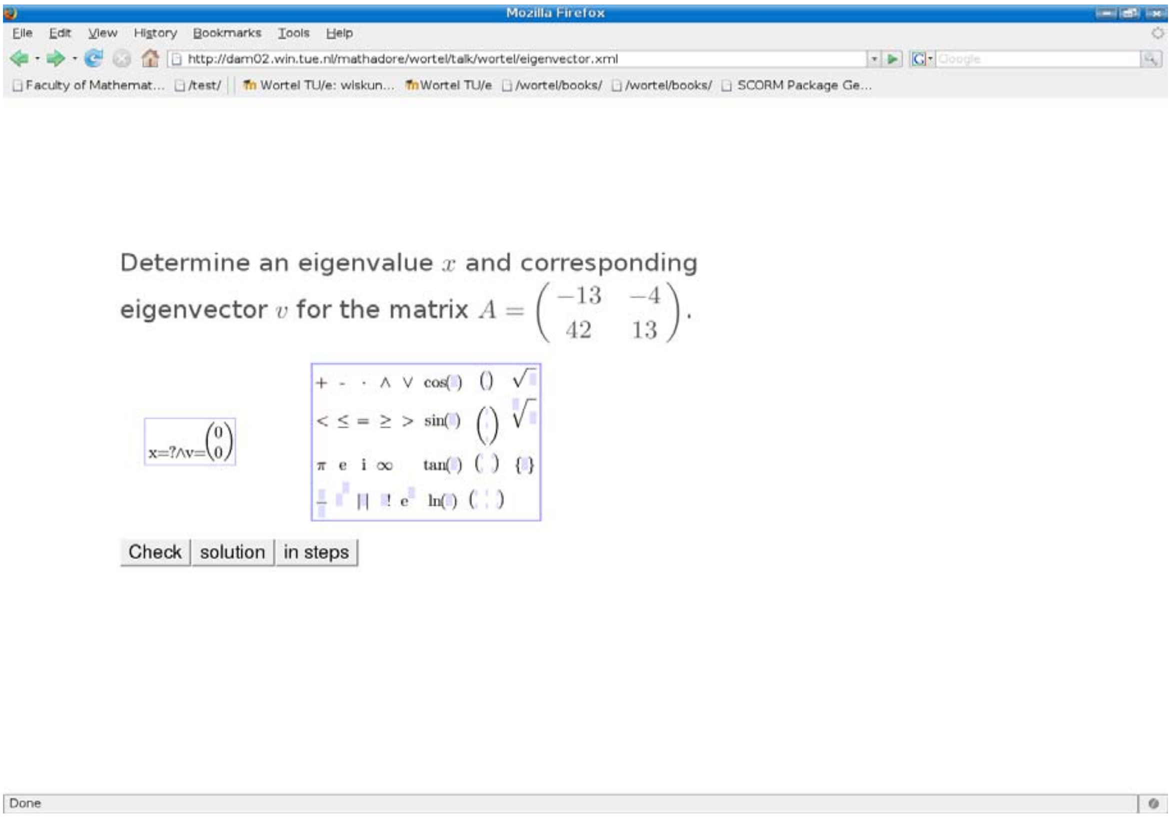

Difficulty levels - Feedback selected
Nr students
9
7
5
3
1
-1
3
4
2
2
3
4
0
3
6
Easy
Medium
Difficult
Level of difficulty
Correct answer
Worked out problem
Step-by-step

Figure 3


Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://localhost/mathadoreplayer/wortel/books/linear_algebra/linear_map/exercises/en/exercise7.xml
The linear map 𝒜 : ℝ³ → ℝ³ satisfies
𝒜((3, −6, −9))=(−30, −36, −24)
𝒜((−3, 9, 36))=(93, 126, 108)
𝒜((2, −4, −3))=(−14, −15, −7).
Find the matrix of 𝒜.
Check your answer
Done

(a)


Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://dam02.win.tue.nl/mathadore/wortel/talk/wortel/eigenvector.xml
Determine an eigenvalue x and corresponding
eigenvector v for the matrix A = (−13 −4; 42 13).
Check
solution
in steps
Done

(b)

Figure 4

Find the inverse of the matrix:

$$\begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 2 \\ 1 & -2 & 3 \end{pmatrix}$$

(a)

‘given three differentiable real functions *f, g* and *h* with derivatives satisfying

$f'=f+g+h$,

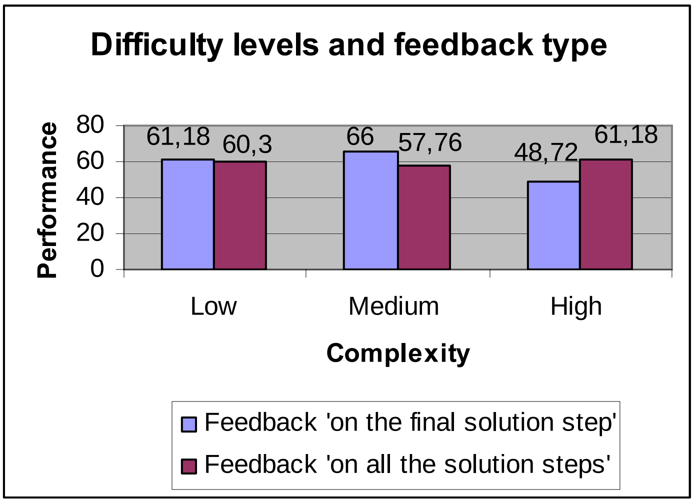
$g'=2g+h$, **and**

$h'=2f-2g$,

is it possible to write the function *k* with $k(x)=1$ for all real *x* as a linear combination of *f,g* and *h*?’

(b)

Figure 5


Difficulty levels and feedback type
Performance
80
60
40
20
0
61,18
60,3
66
57,76
48,72
61,18
Low
Medium
High
Complexity
Feedback 'on the final solution step'
Feedback 'on all the solution steps'