From Royaumont to Lyon

From Royaumont to Lyon: Applications and Modelling During the Sixties

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Abstract At the Royaumont Seminar (1959) the *New Math* reform was officially launched. In the decade between Royaumont and the first ICME congress in Lyon (1969), many mathematics educators were involved in actions to facilitate the implementation of the *New Math* reform in their country. The *New Math* advocates were convinced that a deep knowledge and understanding of the structures of modern mathematics was a prerequisite to arrive at substantial applications, but in actual classroom practices the applied side of mathematics was often completely neglected. But already at the Royaumont Seminar there were alternative voices who pleaded for taking the role of applications seriously. We investigate the arguments for integrating applications in mathematics education, as well as the kind of (new) applications that were envisaged, at the Royaumont Seminar and in the decade thereafter, a period which is still less well documented in the history of our field.

1 Introduction

The OEEC Seminar, held from November 23 to December 5, 1959 at the *Cercle Culturel de Royaumont* in Asnières-sur-Oise (France) is considered as a turning point in the history of mathematics education in Europe and in the United States (De Bock & Vanpaemel 2015). As Bjarnadóttir (2008, p. 145) stated: “The Royaumont Seminar can be seen as the beginning of a common reform movement to modernize school mathematics in the world”. Or in the words of Skovsmose (2009, p. 332): “After the Royaumont seminar, modern mathematics education spread worldwide, and dominated a variety of curriculum reforms”. The famous slogan “Euclid must go!”, launched at Royaumont by the *Bourbakist* Jean Dieudonné, became a symbol of the radical modernization of school mathematics. Most of the Royaumont proposals were strongly influenced by *Bourbaki*, the French structuralist school whose members or adherents, such as Gustave Choquet, Jean Dieudonné, Lucienne Félix and Willy Servais, were well represented at the Seminar. According to these scholars, the basic model for modernizing school mathematics should be the academic discipline of mathematics, as re-constructed and formalized from the late 1930s on by *Bourbaki*. Recent archival research by Schubring (2013) revealed clear-cut evidence for a close connection between the organisers of the Seminar and the entire *Bourbaki* group.

Less well known is that also alternative reform proposals, emphasizing the role of applications in school mathematics, were voiced at Royaumont. This was also inspired by the developments in the field of applications during WW2. These proposals were however less decisive for developments during the 1960s than the dominant structuralist ideas. These ideas, especially what should be taught about the (axiomatic) basis of mathematics in school determined the debate. In this chapter, we first discuss the kind of (new) applications that were envisaged by some Royaumont lecturers, as well as their pleas for integrating the applied side of mathematics in secondary school curricula. This discussion will be based on *New Thinking in School Mathematics* (OEEC 1961a), the official report of the Royaumont Seminar. Second, we examine the views of the more radical *New Math* reformers on applications or more generally, on the usefulness of mathematics. Third, we follow the debate on applications and modelling in the mathematics education community between Royaumont and the first ICME congress in Lyon (August 24 - 31, 1969). For several reasons, this decade is less well documented in the history of our field. At that time, *L'Enseignement Mathématique*, the only international journal on mathematics education until the late 1960s, had become a purely mathematical journal (see Furinghetti 2009) and the international conference series which are now strongly established in our field (ICME, PME, ICTMA, HPM, …) had not yet started. An exception might be the annual meetings of the *International Commission for the Study and Improvement of Mathematics Teaching* (CIEAEM), founded in the early 1950s, but CIEAEM only started publishing *Proceedings* of their meetings in 1974 (Bernet and Jaquet 1998).

Although there were no strong international communication channels in the mathematics education community during the early- and mid-1960s, it was a very rich period of noteworthy international activity, including several seminars and symposia organised under the auspices of OEEC/OECD, UNESCO or ICMI (see, e.g., Furinghetti et al. 1998). These meetings mainly focused on issues related to the forthcoming introduction of *New Math* (program development, renewal of geometry teaching, teacher (re-)eduction and new didactical methods), but occasionally, concerns and proposals about the integration of applications in the curricula were expressed too. On the basis of the *Proceedings* and other edited documents from these meetings, we more generally review the visions at that time on the role of applications in mathematics education at the international forum.

By the end of the 1960s the debate on applications and modelling gained momentum. Hans Freudenthal, at that time president-elect of ICMI, organized the international colloquium "How to Teach Mathematics so as to Be Useful" (Utrecht, August 21-25, 1967). The contributions to that colloquium were published in the first issue of *Educational Studies in Mathematics* (May, 1968). In his introductory address, Freudenthal took the opportunity to explain his views on the colloquium's theme. He argued that students could not be expected to (be able to) apply the mathematics they had been taught in a purely theoretical way. Instead, for enabling students to apply the mathematics they have learned, mathematics education should start from concrete contexts and patiently return to these contexts as often as needed (Freudenthal 1968). It is the beginning of a new era in which applications and modelling gradually became an essential part of mathematics education. In the Netherlands, the theory and practice of *Realistic Mathematics Education* (RME) were developed and inspired the teaching of mathematics in a large number of countries worldwide (Van den Heuvel-Panhuizen et al. in press). We conclude this chapter with a more detailed discussion of Freudenthal's ideas on applications and modelling in the years preceding ICME-1.

Although the focus of this chapter is on what happened in Europe and North America, we are convinced that it is also applicable to most other countries.

2 Applied Mathematics at the Royaumont Seminar

The Royaumont Seminar was organised by the *Office for Scientific and Technical Personnel* (OSTP) of the *Organisation for European Economic Cooperation* (OEEC; later joined by nations outside Europe to form the *Organisation for Economic Co-operation and Development*, OECD). The *Office* was created for the purpose of promoting international action to increase the supply and improve the quality of scientists and engineers in OEEC countries (OEEC 1961a). The main motive for OEEC/OSTP to organise a Seminar aimed at upgrading mathematics education was clearly economic: industry and other branches of economic activity were confronted with new applications of mathematics leading to a demand for more mathematicians with new kinds of skills. Therefore a re-appraisal of the content and methods of school mathematics was needed. In his opening address, Marshall H. Stone, at that time president-elect of ICMI, formulated the functional argument as follows:

> […] the usefulness of mathematics in practical matters has been an added factor in its vitality as a component of the school curriculum. In this period of history it is the rise of modern science and the ensuing creation of a technological society which compels us to give increasing weight to the utilitarian arguments for the more intensive teaching of mathematics (OEEC 1961a, p. 17).

Stone also emphasized the need for a better coordination between mathematics and science teaching: "It is not going to be sufficient to improve the mathematical curriculum as an isolated part (…). It is of the first importance that instruction in mathematics and in the various sciences should be adequately co-ordinated" (OEEC 1961a, p. 21).

In view of the above, the Royaumont Seminar should thus have been a breakthrough of an applied and interdisciplinary perspective in mathematics education, but it turned out differently. Due to a dominance at the Seminar of professional mathematicians, most of them members or adherents of the French structuralist school, pure academic mathematics was *de facto* adopted as a model for school mathematics and most participants only paid lip service to the active application of mathematics. For instance, Dieudonné admittedly referred to applications to theoretical physics as a main argument for the inclusion of new topics in university courses of analysis, but leaved open the question whether any kind of "applied mathematics" should already be integrated in the secondary school programs. Nevertheless, he believed that a favourable consideration of his reform proposals, having a clear *Bourbaki* orientation, would already provide the theoretical foundations for teaching questions of applied mathematics (OEEC 1961a).

An alternative voice at Royaumont was that of Albert W. Tucker, a Canadian mathematician at that time working at Princeton. Tucker discussed the aspect of new uses of mathematics and their implication for mathematics education. Rather than study problems which involve two variables – or at most three or four – as most problems in classical physics, new branches of mathematics are developed to deal with complex realities involving several variables, which often occur in in the social sciences, for instance in economics and psychology. Within these realities, Tucker distinguishes problems of *disorganised complexity* and problems of *organised complexity*. The first category refers to problems with numerous variables and asks for techniques of probability theory and statistical interference, being effective for describing "average behaviour". Problems of *organised complexity* involve a sizable number of factors which are inter-related into an organic whole and require, among other things, a knowledge and use of matrix algebra.

Tucker exemplifies this last category with a problem of linear programming, utilising inequalities, intersections, graphic methods, and unique algebraic procedures for solving equations. According Tucker, an integration in all secondary-school programs of these newer types of mathematics, in a suitable form, is feasible and desirable. He however acknowledges that an effort is needed to enhance teachers' knowledge about modern mathematics and its applications to teach the subject well (OEEC 1961a).

Tucker's plea for the integration of probability theory and statistics in secondary-school curricula was supported by Luke N. H. Bunt from Utrecht University (The Netherlands). Bunt presented at Royaumont the outline of a syllabus on this subject matter taught in a Dutch experiment for the alpha streams of secondary schools (for more details on this experiment, see, e.g., Bunt 1959):

> (a) Some elements of descriptive statistics, such as frequency distributions, histograms, mean, median, and standard deviation.
>
> (b) "Classical" probability theory, with proofs of some of the elementary theorems.
>
> (c) Intuitive treatment of binomial probability distributions; application to physics.
>
> (d) Testing of a hypothesis (Bernoulli type of distribution); null hypothesis; level of significance; sample space; critical region; confidence limits; sign test; rank correlation. Only Type I errors (accepting a false hypothesis) are considered.
>
> (OEEC 1961a, p. 91)

For Bunt the problem of estimating some characteristics of a population on the basis of the values of these characteristics of a sample should be the dominant objective of course in statistics. Also Bunt's proposal went again the general trend of the Royaumont Seminar because (a) he did not primarily focus on those mathematically gifted students that would become mathematicians or engineers, but on future students in economics, psychology and other social sciences, and (b) his didactical approach was pragmatic rather than mathematically rigorous (see also De Bock and Vanpaemel 2015).

3 *New Math* reformers' view on applications and modelling

Although at Royaumont and in the decade thereafter, there were several calls for mathematical instruction to take applications of mathematics seriously (Niss et al. 2007), *New Math*, strongly focussing on theoretical academic mathematics, was – at least in continental Europe – the dominant reform paradigm. Originally, the ambitions of the *New Math* reformers and practitioners' call for a focus on useful mathematics were not in contradiction, or as he stated by Niss (2008):

> It is worth noticing that despite the strong theoretical orientation of the *New Math* movement, its founders insisted that one of the points of the reform was to provide an ideal platform for dealing with the application of mathematics to matters extra-mathematical (p. 72).

Claims about the omnipresence and increased usability of (modern) mathematics can be found in many contemporary sources. In the *Charte de Chambéry*, a main French reform document prepared by the "Commission Lichnerowicz" and adopted by the *Association des Professeurs de Mathématiques de l'Enseignement Public* (APMEP), the broad usability of modern mathematics is emphasized (and used as a main argument for the reform of mathematical teaching at all educational levels).

> Contemporary mathematics is useful in many fields: theoretical physics of course, but also computer science, operational research, stock management of companies, organization charts of big administrations, planning for major projects, sociology, linguistics, medicine (diagnosing), pharmacy ... (Charte de Chambéry 1968).

Georges Papy, the architect of the new mathematical curriculum in Belgium and president of the CIEAEM during the mid-1960s, wrote in the *Préface* of *Mathématique Moderne 1* (MM1), the first volume of his pioneering textbook series:

> The scope of the subjects studied in the first 13 chapters [sets and relations] goes *far beyond the framework of mathematics* [emphasis added] and is, in fact, an introduction to rational approaches commonly used in all fields of thought, science and technology (Papy 1963, p. vii).

A closer look at the MM's approach reveals that Papy indeed occasionally leaves the pure mathematical path and presents "daily-life" situations to introduce new mathematical concepts and structures, but these situations do not incorporate realistic or authentic problem situations to be solved with mathematical tools. Their only purpose is to facilitate comprehension of an abstract formalized definition of the mathematical concept or structure that is targeted. Moreover, the newly learned mathematics is never (re)invested to analyse and to solve new challenging problems outside mathematics.

To better characterize the role of extra-mathematical situations in *New Math* courses of the 1960s, Hilton's (1973) distinction between *illustration* and *application* might be helpful. The point Hilton made is essentially the following. A situation, within or outside mathematics, is an *illustration* of a mathematical theory if and only if that situation clarifies the theory. A situation is an *application* of a mathematical theory if and only if that situation is clarified by the theory. For the high-order mathematical structures of the *New Math*, such as groups, fields or vector spaces, no *applications* were available for the early-aged students to whom these structures were taught and thus, these structures only could be *illustrated* with concrete instantiations (e.g. concrete materials or games especially constructed for that purpose). Although *New Math* advocates often referred to the universal applicability of the powerful structures of the modern mathematics in today's science and technology, they were unable to demonstrate this applicability to their students, for them it were just words, no tools for real problem solving, application analysis or modelling.

> [Mathematical] structures are great and admirable machines, but they can produce, in early mathematics education, only too small things and too small effects. These small things are the naive examples of structures which embellish modern mathematics textbooks and which have been designed especially for students (Rouche 1984, p. 138).

4 Applications and Modelling in the Post-Royaumont Era

Based on a survey of 21 national reports, Kemeny (1964) observed that the main interests of the international mathematics education community in late 1950s/early1960s were one-sidedly directed towards pure mathematics. The debate was focussed on the type of new mathematical subjects that could find a place in secondary school programs, on how the teaching of traditional topics could be improved by the adoption of modern ideas, on the "right" way of teaching geometry, … With the exception of a widely supported plea for teaching some notions of statistics at the secondary level, relatively little attention was paid to the applied side of mathematics. Also at the international meetings on mathematics education of the 1960s, organised by OEEC/OECD, UNESCO or ICMI, only occasionally ideas for integrating applications of mathematics in secondary school curricula were voiced. In the next paragraphs we briefly discuss three main sources of applications that, aside from statistics, were mentioned at these forums.

First, reference was still made to applications of mathematics to classical physics. The *Group of Experts*, that met in Dubrovnik (1960) for the purpose of preparing a detailed synopsis for modern secondary school mathematics, as stipulated in one of the Royaumont resolutions (OEEC 1961a), re-insisted on the need for a better coordination between the teaching of mathematics and the teaching of science (particularly of physics), but provided little or no concrete suggestions to put that coordination into practice. An exception might be the early introduction of vectors and the systematic development of their algebraic and geometrical properties in a modern curriculum for school geometry, which they considered, at least potentially, of the greatest use to the students and teachers of physics (OEEC 1961b). From physical scientists, an increasing pressure was felt to teach a more or less intuitive introduction to calculus in secondary schools – which was not the case in many countries – but mathematics education reformers of that time did not have clear ideas how such introduction could be properly integrated in a modern mathematical curriculum (Kemeny 1964).

Second, there is the mathematics related to the upcoming computing machines which began to fundamentally impact secondary school mathematics. Examples of new computer-related applications and their curricular impact were thoroughly discussed at the OECD conference in Athens (1963). That conference provided a special section on “applications in the modernisation of mathematics” (OECD 1964) in which Henry O. Pollak (USA), one of the pioneers in the field of applications and modelling in mathematics education, examined, among other things, new areas of mathematics motivated by computer sciences. He stated that the basic notions of programming, including the use of flow diagrams in the construction of algorithms, should be essential parts of secondary school curricula. But as Hermann Athen, a German contributor to that Athens conference argued, computers may have a much broader impact:

> A factor not to be neglected is the technical and economic revolution which is taking place as a consequence of the big automatic computers. This revolution in psychic and intellectual functions of human thinking and computing is continuously leading to new investigations in the fields of logic and the analysis of thinking. There is practically no field of mathematical investigation which is not dependent upon the use of computers, e.g. many problems of the social, behavioural, managerial and economic sciences (OECD 1964, p. 245).

Other topics, in some way or another related to computers or their use, were suggested at that time, for instance binary representations of numbers, coding, numerical analysis, discrete mathematics, electrical circuits, logic and Boolean algebra.

Third, as a genuine application to economics and other social sciences, linear programming was repeatedly mentioned. The topic fitted well within a modern course of linear algebra, but also could strengthen students’ numerical skills related to solving equations and inequalities. Moreover, it opened a window to operational research, a recent field of applied mathematics that deals with the application of advanced analytical methods to help make better decisions given certain constraints. Probably more than other fields of application, optimization involves mathematizing and modelling, i.e. interpreting a real-world situation in terms of a precisely formulated mathematical model (OECD 1964). Modelling and models were not yet widespread notions during the 1960s, but they gained ground. In his *Introduction* to the *Proceedings* of the UNESCO colloquium in Bucharest (1968), Nicolae Teodorescu observed that the notion of *model* had acquired universal presence and circulation, and already acknowledged the cyclic nature of modelling processes.

> Modelling the complex, heterogeneous reality is the deliberate aim of any modern research method in sciences of nature, in social sciences and in humanities. The victorious penetration of mathematics in other scientific domains is accounted for by modelling which, repeated successively, leads to mathematical models (International UNESCO Colloquium 1968, p. 27).

5 Freudenthal and the Emerging RME Movement

The end of the 1960s was characterized by an increased interest for the didactics of mathematics, particularly at the micro level. Not only the purely mathematical subjects, but the way a child learns, became a main guiding principle for developing mathematics education. This new trend was reflected in a growing number of (international) congresses and meetings in the field. In the context of this chapter, the ICMI colloquium initiated by Hans Freudenthal around the theme "How to Teach Mathematics so as to Be Useful" (Utrecht, August 21-25, 1967), deserves our special attention. It was the first international meeting in which an international panel discussed the differences in opinion about the role of the use of mathematics (La Bastide-Van Gemert 2015). In his opening address Freudenthal sketched, in a general way, his views on mathematics education. He explained that teaching mathematics "so as to be useful" is not the same as teaching useful mathematics:

> Useful mathematics may prove useful as long as the context does not change, and not a bit longer, and this is just the contrary of what true mathematics should be. Indeed it is the marvelous power of mathematics to eliminate the context. […] In an objective sense the most abstract mathematics is without doubt the most flexible. In an objective sense, but not subjectively […] (Freudenthal 1968, p. 5).

He further argued that we should neither teach "applied mathematics", nor "pure mathematics" (and expect that the student will be able to apply it later). Mathematics is rather learned by doing, as a human activity, as a process of mathematizing reality and if possible, even of mathematizing mathematics.

> The problem is not what kind of mathematics, but how mathematics has to be taught. In its first principles mathematics means mathematizing reality, and for most of its users this is the final aspect of mathematics, too. For a few ones this activity extends to mathematizing mathematics itself (Freudenthal 1968, p. 7).

Freudenthal's colloquium is sometimes regarded as the symbolic beginning of a new era in the history of applications and modelling in mathematics education, the so-called *advocacy phase* (Niss et al. 2007) in which advocates of applications and modelling provided arguments in favor of the serious inclusion of such components in the teaching of mathematics. In several countries (such as, for example, the UK and the US), this phase was quickly followed by a second *development* phase, mainly characterized by the development of new educational materials to put such teaching into practice, sometimes by institutes especially created for that purpose. In the Netherlands, for example, the implementation was driven by the *Institute for the Development of Mathematics Education* (IOWO) – founded in 1971 by Freudenthal, nowadays the *Freudenthal Institute* – which shaped the philosophy and practice of *Realistic Mathematics Education* (RME).

When in Lyon (1969) the first International Congress on Mathematics Education (ICME-1) was held, Freudenthal was president of ICMI. During ICME-1 two of the twenty published contributions were about applications and modelling (Editorial Board of Educational Studies in Mathematics 1969; Gispert 2003, p.262)

6. Conclusion

The *New Math* movement, launched at the Royaumont Seminar, was dominated by academic mathematicians who had a genuine interest in education, but most of them were rather involved in pure than in applied mathematical research. Applications were not their first concern. Moreover they were convinced that a thorough insight in the structures of the *New Math* was a solid and necessary basis for teaching questions of applied mathematics. In daily-school practice *New Math* adherents rather used real-world situations to illustrate than to really apply mathematical structures. During the 1960s calls for taking applications – and later also *mathematical modelling* – seriously grew louder and culminated in Freudenthal's colloquium "How to Teach Mathematics so as to Be Useful". This meeting marked the beginning of a new and more favorable period in the history of the teaching of mathematical modelling and applications.

In sum, it can be stated that the teaching of applied mathematics was not a primary concern during the early- and mid-1960s. But although most leading reformers of that time focussed on pure mathematics and only paid lip service to the active application of mathematics, some mathematics educators highlighted the role of (new) applications – and later also that of modelling – and regarded it as an essential element in the modernization of mathematics teaching.

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