Comenius Open Maths project

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Summary

Teaching mathematics-related topics at universities, I notice two important yet contradictory aspects. STEM (science, technology, engineering and mathematics) subjects require deep mathematical understanding and actual problem solving skills. However, university students often lack proper mathematical knowledge, using instead cookbook-like recipes to solve problems. Consequently, the skill set that students learn from mathematics throughout their education is often inappropriate for their later careers.

Open Maths, as I call it, is a new collection of teaching methods. It is based on state-of-the-art research results from neurosciences, psychology and mathematical didactics (mathematical mindsets), and widespread experiences at lower-level education. The objective of this project is to innovatively adapt these results to online education at a university level. The main pillars of Open Maths are (1) demonstrating positive messages about mathematics and individual potential; (2) developing mathematical sense making by low-entry, high-ceiling problems and by great online resources; (3) establishing an encouraging atmosphere for effective teamwork.

This project involves the development, teaching and evaluation of an entire Open Maths course at the Open University. The purpose of this (optional) course is to prepare students for learning mathematics. In particular, by the end of the course, students will be able to appreciate real mathematical thinking, to reason about logical steps mathematically, to use online resources and software tools, to have courage to learn difficult topics and to recognise phases of mathematical problem solving when collaboration can help. In

summary, the goal is to open mathematics for students and to open students towards mathematics.

Formulation of the problem

As technology is developing at an ever increasing pace, new kinds of learning and teaching strategies are necessary, especially in the higher education and beyond. Technology changes the job market. Old jobs become obsolete as computers are taking over the tasks from humans and new jobs are rising to satisfy novel needs. Future generations need to find their places in this volatile environment with willingness to develop and with life-long learning capacities (Van Merriënboer & Kirschner, 2012). Not only the demand changes on the job market, but the possibilities that the Internet, a flagship technological invention itself, provides. New communication means allow students to be always connected, MOOCs give great opportunities to learn based on intrinsic motivation. Also, great resources, including tutorials and catchy videos, are freely available. All these advances are intertwined with mathematics. The new jobs hold problems that we cannot yet define, requiring mathematical openness and flexibility (National Mathematics Advisory Panel, 2008). YouTube is full of videos that offer lovely material for those who are willing to think and embrace the creative nature of mathematics. These resources are just too good to not include them in future higher education and life-long learning. The only missing element is to release students' own potential to learn real mathematics. That is what Open Maths provides.

Being originally a mathematician and maths teacher and having taught for several years, I made a number of interviews with computer science bachelor students about their experiences with mathematics in education and elsewhere. My goal was to find out why students often cannot solve problems that deviate from typical or learnt exercises. These interviews revealed such words as speed (‘snelheid’), roadmap (‘stappenplan’) and solitary (‘eenzaam’) associated with mathematics.

Mathematics for them is utterly structured and boring; creativity and exploration are concepts that they cannot link to this subject. In contrast, PhD students in the same discipline report about a very different mathematics, which is joyful, full of connections, discoveries, and in which collaboration is of great importance. Mathematical problem solving in real-life situations is closely related to the second meaning of mathematics (National Mathematics Advisory Panel, 2008).

A small experiment will make it easier for me to explain important aspects in this proposal. I invite therefore the reader to participate. Please stop for a moment, take your time, and compute the following simple multiplication in your head: 8·15. If you are ready, please proceed.

Think about how you computed it, what the steps were you took. Did you go like (10·15) – (2·15)? Or (8·10) + (8·5)? Perhaps (8·5)·3 or (2·15)·4? Or some other way? The interesting thing is that people choose different strategies (Boaler, 2016). Automatically, not consciously. The chosen one is the natural for them. People think differently. That is often useful, because mathematics is a creative and personal subject, even at this most basic level. And as we move towards more involved topics, it becomes increasingly more flexible. Furthermore, because of these differences, it is well worth collaborating. When a complicated problem has to be solved, everybody can contribute.

Another aspect of this experiment is to demonstrate number sense. A number can be viewed in many ways, depending on its current role. Sometimes it is good to see 8 as (10 – 2), sometimes as (4·2) or 2^3. Research shows that students who do not think with number flexibility, have difficulties throughout their mathematical studies (Gray & Tall, 1994, Feikes & Schwingendorf, 2006).

A third aspect is that such simple problems can be used to encourage students. When I tried this experiment in class, a student calculated it

as ((15·2)·2)·2. He thought it was childish because he tried to avoid the times table; he thought doubling was much simpler. In the contrary, this was one of many creative solutions; this one being similar to the way a computer computes. Analysing such a solution can reveal various things, including the unconscious use of commutativity. It can also be visualised with rectangles (similarly to other strategies). Visualisation, like various representations, is essential in mathematics and students hardly ever use it. Moreover, doubling makes memorisation redundant and it is easier to generalise. Generalisation is another crucial step in higher-level mathematics.

Students who think that their thinking is childish or who associate mathematics with memorisation and just following boring methods will not engage in solving challenging problems (Gray & Tall, 1994).

Recent neuroscience and psychology research shows important facts about our learning abilities. Neuroplasticity is the brain's ability to change throughout our lives so that our mental capacities can be extended (Draganski, Gaser, Busch, Schuierer, Bogdahn, & May, 2004, Woollett & Maguire, 2011). The research of Stanford psychologist Carol Dweck (Dweck, 2006) has demonstrated that people think differently about self-development. People with a fixed mindset believe that their intelligence and other talents are gifts by birth, whereas other people with a growth mindset focus on their progress. One's mindset has a profound effect on learning. Focussing on talent, one chooses strategies that are well-known for him/her and avoids challenges. This is because a hard problem that cannot be solved instantly sends an implicit message to this person that he/she is not talented in that area. On the other hand, focusing on growth, one is willing to be challenged as he/she discerns such situations as a chance to develop. Dweck has also discovered that a fixed mindset can be changed to a growth mindset to spur successful learning. She discusses various contexts with regard to mindsets including sports, management and relationships. So, one can develop even as an adult.

Similarly, growth mindset can also be encouraged in mathematics (Boaler, 2016). Students who seek challenges and believe that they can evolve, will be able to learn. They will be able to think deeply, to discover patterns and relations, to put forward conjectures and to test and prove them. These skills (Polya, 1957, Drijvers, 2015) are beneficial in the typical university subjects (e.g., calculus, discrete mathematics, linear algebra, probability and statistics) for students. But more importantly, they are useful in their career, which takes place in a challenging future, full of uncertainties, new professions and ever improving computing capability and artificial intelligence.

For centuries the joy of mathematics, proper sense-making of mathematical problems and a deep understanding of mathematical concepts and relations have been a privilege for just a selected group of people. Because of research results from psychology, neuroscience and teaching, mathematics now becomes accessible for many more. This is important not only as university students are required to progress efficiently in their studies using the ever changing and developing technological environment, but also because future generations will face problems that are not yet even predictable. Therefore, Open Maths may be a disruptive innovation in higher education that is desperately needed for our students.

Innovation and objected results

In this project, an Open Maths introductory course is developed and taught at the Open University (OU). This will be the first growth-mindset mathematical university course that takes place in The Netherlands; and to the best of our knowledge, only the second in the world after the Stanford University, which launched a course for future engineers just now as I started writing this proposal. (We acknowledge that other introductory courses to mathematics and mathematical thinking exist in The Netherlands, such as (“Mathematics preparation,” 2017).)

Open Maths is in many ways based on Jo Boaler's now increasingly more popular Mathematical Mindsets (Boaler, 2016), which provides innovative mathematics teaching for lower education. Open Maths, however, focuses on the university level. It covers higher-level conceptual thinking as well as develops new problems and problem sets. Finally, it incorporates many Internet resources and software tools.

In this project, an eight-week-long, introductory Open Maths course is designed, carried out and evaluated. The course's goal is to evolve positive attitude in students toward learning mathematics. Besides focusing on growth mindset, we make maximal use of the online platform by including great resources (see *e.g.* the experimental problem set 5) and by letting students collaborate. As a result, participants will be able to reason about mathematical steps, think deeply about and explore new problem situations, and will value the ideas of others and those of their own. Students change their perception from a method-oriented mathematics to an open, creative mathematics, which will help them be successful in their science and engineering education, and later in their career.

This project is intended as a first step to introduce Open Maths in The Netherlands.

Project plan

The goal of this project is to create the first Open Maths course (for 15-50 students), which can then be later repeated after some possible improvement, adapted and spread to other institutes and possibly further developed into a widely available MOOC. To achieve this, the course is very thoroughly prepared based on research, experience and the OU's online learning environment. We plan to execute the project following four work packages (WPs).

WP1. Course design and preparation: Administrative tasks of the preparation include the writing of general descriptions (website, flyer, ...) and the course syllabus and the arrangement of the technical

background at OU. The course design tasks comprise research, the collection and creation of further suitable problems and problem sets, the planning and the organisation of the eight sessions and the creation of the course material, including presentations. It is worth noting that a mathematical biography and a journal will be an important part of the homework. In the former, students collect their earlier experiences with mathematics, in the latter, they can reflect on their own learning process throughout the course. Finally, the OU learning environment 'yOUlearn' has to be prepared by filling up content and make the communication interfaces (discussion folders, meetings) ready.

WP2. Course run: Besides the weekly preparation, the giving of online classes and providing homework feedback, we actively participate in the discussions. Furthermore, students will do two mathematical mindset tests; one at the beginning of the course and one afterwards. Such a test measures how flexibly and openly students approach unknown problems and problem areas. The tests are evaluated for the students as well as they play an important role in the course evaluation (WP3).

WP3. Course evaluation: The course is evaluated by students using OU's standard course-evaluation method. Besides this, we are also curious to learn students development with regard to Open Maths: flexibility (representations, strategies, reasoning, collaboration) in terms of mathematical problems, growth mindset and openness to new mathematical areas. Therefore, the biographies and the journals are summarised and the above-mentioned tests are analysed. Moreover, an in-depth interview is made with at least five attendants after the course an academic quarter later. An important question is how students could apply their new attitude and skills in mathematics-related courses. A report is produced about the most interesting findings for publication.

WP4. Dissemination: The goal of dissemination is to make the higher-education teaching community within and outside the OU aware of the potential of Open Maths. This project is a first step to gain personal and

measurable experiences of an entire course life-cycle. Further details are described in the 'Result for the teaching community' section.

Result for the teaching community

Need for Open Maths introductory courses. Last year at the Open University, 178 students registered for Linear Algebra, 63 of those attempted the exam and 47 passed. This is less than 27% of all students, even though those who undertook the exam succeeded with 75%. There is certainly a need to give more chance to 115 students who do not have the courage and skills to take the exam. (Other first-year mathematics courses show similar statistic.) Although the student population at OU has very diverse background knowledge and objectives, which partly explains this huge gap, the OM course will very likely improve the situation.

Other Open Maths courses at OU. After the success of this project, we plan to apply the Open Maths's approach to higher-level courses at the OU. A course can entirely be redesigned in the spirit of Open Maths. But even if a course does not change substantially, some aspects of Open Maths can be adapted: 1. continuous positive message about mathematics and students' potentials to motivate perseverance; 2. some problems can be opened up to make them more exploratory in nature to let students improve their problem-solving abilities; and 3. collaboration can be encouraged at more complex problems to nurture other (*e.g.* social, reasoning) skills and to enhance effectiveness. Moreover, a MOOC for university students is among the internal plans at the Open University. (The first step in this direction is the publication of the entire course material from this project.)

OM outside the OU. We foresee two directions after this project. First, other universities and *HBO*s will possibly overtake an introductory Open Maths course or some of its influencing ideas. According to my experience, currently, only a small group of students can in general be effective in later mathematics-related subjects and choose challenging

(*e.g.* bachelor, intern, master) projects. The more its students succeed, the better for an institute. Second, the Open Maths approach will probably effect higher-level mathematics courses too. (As mentioned above, we will work on that within the OU.)

Lower-level education. We intend to collaborate with lower-level education in the long run. Research with mathematical didactic groups in the country, talks and workshops at teachers' education and trainings at schools are all desirable options.

Professional Statement

Teaching itself is changing; the teacher's underlying motivation is constant. As information and communication are becoming ubiquitous, an educator's role is not the most straightforward source of knowledge anymore for students. However, the educator still wants to convey the joy of learning and understanding.

I always believed that the best starting point for learning is internal motivation. If you are eager to grasp something then your mind becomes open to that. To motivate students, nevertheless, is often difficult as today's students are flooded with sensational stories and distracted by short messages and tweets. In class teachers have to convince students to pay attention, yet they have a direct overview of the situation. In online education this is different; the teacher has even less control.

Recently, I started applying Dweck's idea of growth mindset in teaching, which has given a new boost of stimulating internal motivation. If students are convinced that they are able to develop, then they change their learning habits and put much more effort in their work. For instance, in one of my (online) courses, students have to write an essay in two steps. About the first version I give them feedback, in which I provide constructive advice for improvements with growth-mindset messages (focusing on: improvement, belief in the learner's

abilities and further perseverance). The second, final version is marked according to a rubric. The result is amazing: Most students write very good essays, which reflect a lot of work, enthusiasm and effort to write well. The average of the marks after the strict evaluation is more than 9!

I am a mathematician and teacher according to my first Master's degree. I always loved teaching and learning. I have followed several MOOCs in the last five years. And for many years I have taught mathematics, computer-science and software-engineering courses at universities in class and online. I am convinced that sciences and engineering, in general, require skills with regard to deep mathematical thinking. *In our highly computerised world, students have to prepare for a career, in which they have to compete not only with other humans but also with systems.* While computers are better at computation and following methods, students need to develop confidence, analytical skills, efficient collaboration skills and critical thinking (Brynjolfsson, McAfee, 2011, Arai, Matsuzaki, 2014). And most importantly, they need to enjoy activities related to mathematics and learning because people need to evolve throughout their lives.

How can I achieve this? Open Maths is an approach to applying growth mindset in mathematics teaching at the higher education. As mentioned above, feedback and the messages received by the learner have a huge impact on students' effort and results. Observing a Masters course for future teachers by prof. Jo Boaler at the Stanford University, I also experienced that this new approach to teaching can bring quite an abrupt change. In a matter of minutes, students start opening up to mathematics and believing in their own capacity of developing, which then further reinforced in the rest of the course. It is a profound experience! In my talks, I always encounter the same impact. The audience starts working, collaborating and finding creative strategies to maths problems.

Now it is time to adapt this to online education. Not only because I work at the Open University, primarily providing distance education, but also

because it has a potential to reach a wide audience. Therefore, this Fellowship provides the opportunity to start changing the image of mathematics. I can develop an online course, which supplies university students with the appreciation of actual mathematics, which is full of patterns, relations, creativity and in which fun and useful to work with others. In short, this course prepares them not only for later maths-related courses but also for solving real-world problems later.

The course within this Fellowship is the first step. At the Open University, it can be extended in two ways. First, the course can be given to other students who need a change their mathematical thinking, including bachelor and master students at the entire university (computer science, other sciences and psychology). Second, Open Maths can also start affecting other mathematics subjects. Outside the university, the course can be extended for any students. Furthermore, Open Maths can have an effect also on mathematics teacher education. (See also Dissemination.) Finally, since I intend to focus increasingly more on Open Maths, this project can have a very positive effect on my career.

To set up this course does not lack challenges. Collaboration is very different in a classroom than via the Internet, and it is hard to induce intrinsic motivation. On the other hand, in the online environment, it is most natural to involve other online resources; after all, this environment is the learner's everyday 'habitat'. The challenges have therefore the potential to turn them into opportunities.

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